5: The First-Order Soundfield Microphone

The first-order soundfield microphone represents the “state of the art” in coincident microphone array technology. Its use enables recordings for ambisonic reproduction to be made, by facilitating the direct transduction of live sound fields into first-order B-format. It may also be employed as a coincident pair for stereo recording, in which application it provides a number of unique features.

5.1: Principles of Operation

The first-order B-format signals W, X, Y and Z correspond to the outputs of four coincident microphones; one omnidirectional microphone and three mutually orthogonal first-order pressure gradient microphones. Because it is not possible to position such a set of microphones in an arrangement in which true coincidence is sufficiently well approximated, the first-order soundfield microphone does not have this form. Instead, it consists of a small regular tetrahedral arrangement of first-order microphone capsules. The output signals from these capsules are referred to as A-format signals, and the B-format signals are derived from them by a process of matrixing and filtering.

The tetrahedron is oriented such that the faces, and hence the directivity axes of the individual microphone capsules, point in directions which may be described as left forward up, right backward up, left backward down and right forward down; these are conventionally denoted LFU, RBU, LBD and RFD. The third letter in each of these designations is redundant so far as unique identification of each microphone capsule is concerned, but it is retained here as a matter of convention, and also to avoid confusion with another set of directional designations which will be introduced latter in connection with the second-order soundfield microphone.

Let v_{LFU} be the output signal from the LFU capsule, and etc.; the A-format to B-format signal matrix, or “A-B matrix”, is then defined by the equations [28]

$$\tilde{W} = v_{LFU} + v_{RBU} + v_{LBD} + v_{RFD} \tag{5.1a}$$

$$\tilde{X} = v_{LFU} + v_{RFD} - v_{RBU} - v_{LBD} \tag{5.1b}$$

$$\tilde{Y} = v_{LFU} + v_{LBD} - v_{RBU} - v_{RFD} \tag{5.1c}$$

$$\tilde{Z} = v_{LFU} + v_{RBU} - v_{LBD} - v_{RFD} \tag{5.1d}$$

where the tilde is used to indicate that these are the signals obtained after matrixing but before filtering. It will be demonstrated in section 5.2 that, if the four capsules were genuinely coincident, then the signals $\tilde{W}$, $\tilde{X}$, etc., would, with the application of suitable relative scaling factors, be the required B-format signals *W*, *X*, etc. However, the non-zero spacing of the capsules in practice necessitates filtering to compensate for the effects of phase differences between their outputs, and in particular to ensure that the B-format signals are phase-coincident. Each signal associated with a spherical harmonic polar pattern of the same order requires the same non-coincidence compensation filtering; hence, one filter characteristic will be required for *W*, and another for *X*, *Y* and *Z* [42] [7].

It may be shown that [42] [7], if the four microphone capsules have cardioid polar patterns, then

$$A_0\{\tilde{W}\} \propto j_0(kr) + jj_1(kr) \tag{5.2}$$

where the notation $A_0\{\tilde{W}\}$ should be read as "the value of A_0 in the Laplace series expansion of $\tilde{W}$", and

$$\left.\begin{array}{l} A_1\{\tilde{Z}\} \\ A_{1,1}\{\tilde{X}\} \\ B_{1,1}\{\tilde{Y}\} \end{array}\right\} \propto j_0(kr) + j3j_1(kr) - 2j_2(kr) \tag{5.3}$$

The non-coincidence compensation filters should have frequency response characteristics which are the inverses of these functions. Gerzon suggested the use of filters having the following characteristics [53]:

$$F_0(w) = \frac{1 + jwt - \frac{1}{3}w^2t^2}{1 + \frac{1}{3}jwt} \tag{5.4}$$

for W and

$$F_1(w) = \sqrt{6}\left(\frac{1+\frac{1}{3}jwt - \frac{1}{3}w^2t^2}{1+\frac{1}{3}jwt}\right) \tag{5.5}$$

for each of the first-order signals, where

$$t = \frac{r}{c} \tag{5.6}$$

and r is the distance of each microphone capsule from the array centre, so that

$$wt = kr \tag{5.7}$$

These filter characteristics are rational approximations to the ideal inverse functions; they were not however derived using any of the standard rational approximation techniques [18].

It may be shown that there are no first-order or second-order components in the Laplace series expansion of $\tilde{W}$; that there is no zeroth-order component in the Laplace series for $\tilde{X}$, $\tilde{Y}$ or $\tilde{Z}$; and that

$$A_1\left\{\tilde{X}\right\} = 0 \tag{5.8a}$$
$$B_{1,1}\left\{\tilde{X}\right\} = 0 \tag{5.8b}$$

and correspondingly for $\tilde{Y}$ and $\tilde{Z}$ [42]. These results indicate that there is no crosstalk between the derived B-format signals in the sense that the spherical harmonic associated with the desired directional response of any one signal does not appear in the Laplace series expansion of any other signal.

The B-format signals obtained from the soundfield microphone may be regarded as coincident up to a limiting frequency f_l given by [7] [42]

$$f_l \approx \frac{c}{p\, r} \tag{5.9}$$

In the case of currently available soundfield microphones, r is approximately equal to 1 cm; this allows effective coincidence to be maintained up to around 10 kHz, a much higher limit than is achievable using ordinary "coincident" microphone arrangements [28] [42] [94]. Above this limiting frequency, the directional responses of the derived B-format signals are severely corrupted by unwanted spherical harmonic components. Since there is no precise definition of the degree of corruption by spurious spherical harmonic components which is "severe", so this limit is to that extent arbitrary.

5.2: Analysis

In this section a mathematical model of the first-order soundfield microphone is developed; this model is suitable for the purpose of understanding the matrixing and filtering employed to obtain the first-order B-format signals. A similar model is required to facilitate the design of the second-order soundfield microphone. Although the results given in the previous section have long been available in the literature, no comprehensive account of their derivation has been published. The analysis presented in this section therefore serves to "fill in the gaps" in the existing literature.

We assume a coordinate system with the origin at the centre of the tetrahedron. With each of the four microphone capsules, corresponding to the faces of the tetrahedron, a unit vector is associated which is normal to that face, and therefore defines the directivity axis of the capsule. By considering the geometry of the tetrahedron, we find that these unit normal vectors are

$$\hat{\mathbf{u}}_{LFU} = \frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1 & 1\end{bmatrix}^T \tag{5.10a}$$

$$\hat{\mathbf{u}}_{RBU} = \frac{1}{\sqrt{3}}\begin{bmatrix}-1 & -1 & 1\end{bmatrix}^T \tag{5.10b}$$

$$\hat{\mathbf{u}}_{LBD} = \frac{1}{\sqrt{3}}\begin{bmatrix}-1 & 1 & -1\end{bmatrix}^T \tag{5.10c}$$

$$\hat{\mathbf{u}}_{RFD} = \frac{1}{\sqrt{3}}\begin{bmatrix}1 & -1 & -1\end{bmatrix}^T \tag{5.10d}$$

The position vectors of the capsule centres are then

$$\mathbf{x}_{LFU} = r\hat{\mathbf{u}}_{LFU} \tag{5.11a}$$

$$\mathbf{x}_{RBU} = r\hat{\mathbf{u}}_{RBU} \tag{5.11b}$$

$$\mathbf{x}_{LBD} = r\hat{\mathbf{u}}_{LBD} \tag{5.11c}$$

$$\mathbf{x}_{RFD} = r\hat{\mathbf{u}}_{RFD} \tag{5.11d}$$

The positions and orientations of the capsules are shown pictorially in figure 5.1.

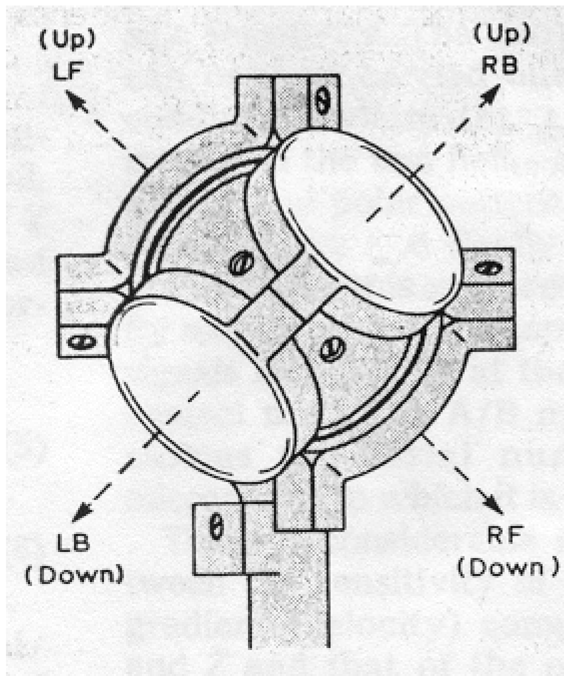

Figure 5.1: Capsule Configuration for First-Order Soundfield Microphone

(reproduced from [28])

As in the analysis of Chapter 3, we consider a plane wave incident from a direction $\hat{\mathbf{d}}$ with wave incidence vector $\tilde{\mathbf{k}}$. Let the sound pressure at the origin - that is, at the centre of the microphone array - be

$$p_O = Ae^{jwt} \tag{5.12}$$

then the pressure at each of the capsules may be written

$$\begin{aligned} p_{LFU} &= Ae^{j(wt+\tilde{\mathbf{k}}\cdot\mathbf{x}_{LFU})} \\ &= p_O e^{j\tilde{\mathbf{k}}\cdot\mathbf{x}_{LFU}} \end{aligned} \tag{5.13a}$$

$$\begin{aligned} p_{RBU} &= Ae^{j(wt+\tilde{\mathbf{k}}\cdot\mathbf{x}_{RBU})} \\ &= p_O e^{j\tilde{\mathbf{k}}\cdot\mathbf{x}_{RBU}} \end{aligned} \tag{5.13b}$$

$$\begin{aligned} p_{LBD} &= Ae^{j(wt+\tilde{\mathbf{k}}\cdot\mathbf{x}_{LBD})} \\ &= p_O e^{j\tilde{\mathbf{k}}\cdot\mathbf{x}_{LBD}} \end{aligned} \tag{5.13c}$$

$$\begin{aligned} p_{RFD} &= Ae^{j(wt+\tilde{\mathbf{k}}\cdot\mathbf{x}_{RFD})} \\ &= p_O e^{j\tilde{\mathbf{k}}\cdot\mathbf{x}_{RFD}} \end{aligned} \tag{5.13d}$$

The A-format signals are given by

$$v_{LFU} = G\frac{1}{a+b}\left[a + b\hat{\mathbf{u}}_{LFU} \cdot \hat{\mathbf{d}}\right]p_{LFU} \tag{5.14a}$$

$$v_{RBU} = G\frac{1}{a+b}\left[a + b\hat{\mathbf{u}}_{RBU} \cdot \hat{\mathbf{d}}\right]p_{RBU} \tag{5.14b}$$

$$v_{LBD} = G\frac{1}{a+b}\left[a + b\hat{\mathbf{u}}_{LBD} \cdot \hat{\mathbf{d}}\right]p_{LBD} \tag{5.14c}$$

$$v_{RFD} = G\frac{1}{a+b}\left[a + b\hat{\mathbf{u}}_{RFD} \cdot \hat{\mathbf{d}}\right]p_{RFD} \tag{5.14d}$$

Note that, since the incident wave is assumed to be a plane wave, so the direction of incidence does not depend on the positions of the capsules.

Substituting the above expressions for the A-format signals in equation (5.1a) gives

$$\begin{aligned} \tilde{W} = G\frac{1}{a+b}\big[&a(p_{LFU} + p_{RBU} + p_{LBD} + p_{RFD}) \\ &+ b(p_{LFU}\hat{\mathbf{u}}_{LFU} + p_{RBU}\hat{\mathbf{u}}_{RBU} + p_{LBD}\hat{\mathbf{u}}_{LBD} + p_{RFD}\hat{\mathbf{u}}_{RFD}) \cdot \hat{\mathbf{d}}\big] \end{aligned} \tag{5.15}$$

A coincident capsule approximation is obtained by setting $r = 0$ (which is equivalent to assuming that *kr* is negligibly small); then

$$\begin{aligned}\tilde{W} &= G\frac{1}{a+b}\left[4ap_O + bp_O(\hat{\mathbf{u}}_{LFU} + \hat{\mathbf{u}}_{RBU} + \hat{\mathbf{u}}_{LBD} + \hat{\mathbf{u}}_{RFD})\cdot\hat{\mathbf{d}}\right] \\ &= G\frac{1}{a+b}\left[4ap_O + bp_O\mathbf{0}\cdot\hat{\mathbf{d}}\right] \\ &= \frac{4Ga}{a+b}p_O\end{aligned} \tag{5.16}$$

Considering now $\tilde{X}$, substitution in equation (5.1b) gives

$$\begin{aligned}\tilde{X} = G\frac{1}{a+b}&\left[a(p_{LFU} + p_{RFD} - p_{LBD} - p_{RBU})\right. \\ &\left. + b(p_{LFU}\hat{\mathbf{u}}_{LFU} + p_{RFD}\hat{\mathbf{u}}_{RFD} - p_{LBD}\hat{\mathbf{u}}_{LBD} - p_{RBU}\hat{\mathbf{u}}_{RBU})\cdot\hat{\mathbf{d}}\right]\end{aligned} \tag{5.17}$$

and the coincident capsule approximation yields

$$\begin{aligned}\tilde{X} &= G\frac{1}{a+b}\left[a(2p_O - 2p_O) + bp_O(\hat{\mathbf{u}}_{LFU} + \hat{\mathbf{u}}_{RFD} - \hat{\mathbf{u}}_{LBD} - \hat{\mathbf{u}}_{RBU})\cdot\hat{\mathbf{d}}\right] \\ &= \frac{Gb}{a+b}(\hat{\mathbf{u}}_{LFU} + \hat{\mathbf{u}}_{RFD} - \hat{\mathbf{u}}_{LBD} - \hat{\mathbf{u}}_{RBU})\cdot\hat{\mathbf{d}}p_O \\ &= \frac{Gb}{a+b}\begin{bmatrix}4/\sqrt{3} \\ 0 \\ 0\end{bmatrix}\cdot\begin{bmatrix}\cos(q)\cos(f) \\ \sin(q)\cos(f) \\ \sin(f)\end{bmatrix}p_O \\ &= \frac{4}{\sqrt{3}}\frac{Gb}{a+b}\cos(q)\cos(f)\,p_O\end{aligned} \tag{5.18}$$

It can be seen from equations (5.16) and (5.18) that, when *kr* is negligibly small, the polar patterns associated with $\tilde{W}$ and $\tilde{X}$ are just those which are required for *W* and *X*, and there is no phase shift between them; equivalent results hold for the remaining first-order signals. Hence, to obtain the B-format signals, it is necessary only to apply any additional gain required to compensate for the differing effective responsivities with which the zeroth-order and first-order signals are captured, and to provide the 3 dB boost of the first-order signals relative to *W* mandated by the B-format specification. If the capsules have cardioid polar patterns, so that $a = b = 1$, then equations (5.16) and (5.18) become

$$\tilde{W} = 2Gp_O \tag{5.19}$$

and

$$\tilde{X} = \frac{2G}{\sqrt{3}} \cos(q) \cos(f)\, p_O \tag{5.20}$$

so that a gain of $\sqrt{6}$ must be applied to the first-order component signals. This is consistent with the low-frequency responses of the filters described by equations (5.4) and (5.5).

In the above discussion, the known matrixing scheme employed in the first-order soundfield microphone has been analysed. A similar method may be used to derive the necessary matrix relationships; this will now be demonstrated, taking the ***Y*** signal as an example. The required polar pattern

$$M_Y(q, f) = KG \sin(q) \cos(f) \tag{5.21}$$

where ***K*** is an arbitrary constant. This may also be written as

$$M_Y(q, f) = KG\, \hat{\mathbf{y}} \cdot \hat{\mathbf{d}} \tag{5.22}$$

Now, we may express $\tilde{Y}$ in the form

$$\tilde{Y} = g_{LFU} v_{LFU} + g_{RBU} v_{RBU} + g_{LBD} v_{LBD} + g_{RFD} v_{RFD} \tag{5.23}$$

where g_{LFU}, etc., are the matrix coefficients to be found. Substituting in the expressions for the A-format signals and making the coincident capsule approximation gives

$$\begin{aligned} \tilde{Y} = G \frac{1}{a+b} \big[& a(g_{LFU} + g_{RBU} + g_{LBD} + g_{RFD}) \\ & + b(g_{LFU} \hat{\mathbf{u}}_{LFU} + g_{RBU} \hat{\mathbf{u}}_{RBU} + g_{LBD} \hat{\mathbf{u}}_{LBD} + g_{RFD} \hat{\mathbf{u}}_{RFD}) \cdot \hat{\mathbf{d}} \big] p_O \end{aligned} \tag{5.24}$$

Since there is no omnidirectional component in the desired polar response, we require that

$$g_{LFU} + g_{RBU} + g_{LBD} + g_{RFD} = 0 \tag{5.25}$$

while equating first-order gradient components yields

$$
\begin{aligned}
\frac{Gb}{a+b}(g_{LFU}\hat{\mathbf{u}}_{LFU}+g_{RBU}\hat{\mathbf{u}}_{RBU}+g_{LBD}\hat{\mathbf{u}}_{LBD}+g_{RFD}\hat{\mathbf{u}}_{RFD}) &= KG\,\hat{\mathbf{y}} \\
g_{LFU}\hat{\mathbf{u}}_{LFU}+g_{RBU}\hat{\mathbf{u}}_{RBU}+g_{LBD}\hat{\mathbf{u}}_{LBD}+g_{RFD}\hat{\mathbf{u}}_{RFD} &= \frac{K(a+b)}{b}\hat{\mathbf{y}}
\end{aligned}
\tag{5.26}
$$

Equations (5.25) and (5.26) may be solved to give

$$g_{LFU}=\frac{\sqrt{3}}{4}\frac{K(a+b)}{b} \tag{5.27a}$$

$$g_{RBU}=-\frac{\sqrt{3}}{4}\frac{K(a+b)}{b} \tag{5.27b}$$

$$g_{LBD}=\frac{\sqrt{3}}{4}\frac{K(a+b)}{b} \tag{5.27c}$$

$$g_{RFD}=-\frac{\sqrt{3}}{4}\frac{K(a+b)}{b} \tag{5.27d}$$

These coefficients do indeed give the desired combination of A-format signals, as may be seen by comparison with equation (5.1c), although we have included additional scaling information here. It will be appreciated that whether these scaling factors are included in the A-B matrix or the non-coincidence compensation filter is a matter of arbitrary choice.

We now consider the derivation of the frequency response functions given by equations (5.2) and (5.3). At this stage we must abandon the coincident capsule approximation, since these frequency responses depend on phase differences between the capsules. Substituting equation (5.14) in equation (5.1a), we obtain

$$
\begin{aligned}
\tilde{W} &= G\frac{1}{a+b}\left[a+b\hat{\mathbf{u}}_{LFU}\cdot\hat{\mathbf{d}}\right]p_{LFU}+G\frac{1}{a+b}\left[a+b\hat{\mathbf{u}}_{RBU}\cdot\hat{\mathbf{d}}\right]p_{RBU} \\
&\quad +G\frac{1}{a+b}\left[a+b\hat{\mathbf{u}}_{LBD}\cdot\hat{\mathbf{d}}\right]p_{LBD}+G\frac{1}{a+b}\left[a+b\hat{\mathbf{u}}_{RFD}\cdot\hat{\mathbf{d}}\right]p_{RFD} \\
&= G\frac{1}{a+b}\Big\{\left[a+b\hat{\mathbf{u}}_{LFU}\cdot\hat{\mathbf{d}}\right]p_{LFU}+\left[a+b\hat{\mathbf{u}}_{RBU}\cdot\hat{\mathbf{d}}\right]p_{RBU} \\
&\quad +\left[a+b\hat{\mathbf{u}}_{LBD}\cdot\hat{\mathbf{d}}\right]p_{LBD}+\left[a+b\hat{\mathbf{u}}_{RFD}\cdot\hat{\mathbf{d}}\right]p_{RFD}\Big\}
\end{aligned}
\tag{5.28}
$$

Substituting from equation (5.13) and leaving the p_O factor implicit gives

$$\begin{aligned}\tilde{W} &= G\frac{1}{a+b}\Big\{\big[a+b\hat{\mathbf{u}}_{LFU}\cdot\hat{\mathbf{d}}\big]e^{j\tilde{\mathbf{k}}\cdot\mathbf{x}_{LFU}} + \big[a+b\hat{\mathbf{u}}_{RBU}\cdot\hat{\mathbf{d}}\big]e^{j\tilde{\mathbf{k}}\cdot\mathbf{x}_{RBU}} \\ &\qquad + \big[a+b\hat{\mathbf{u}}_{LBD}\cdot\hat{\mathbf{d}}\big]e^{j\tilde{\mathbf{k}}\cdot\mathbf{x}_{LBD}} + \big[a+b\hat{\mathbf{u}}_{RFD}\cdot\hat{\mathbf{d}}\big]e^{j\tilde{\mathbf{k}}\cdot\mathbf{x}_{RFD}}\Big\} \\ &= G\frac{1}{a+b}\Big\{\big[a+b\hat{\mathbf{u}}_{LFU}\cdot\hat{\mathbf{d}}\big]e^{jkr\hat{\mathbf{d}}\cdot\hat{\mathbf{u}}_{LFU}} + \big[a+b\hat{\mathbf{u}}_{RBU}\cdot\hat{\mathbf{d}}\big]e^{jkr\hat{\mathbf{d}}\cdot\hat{\mathbf{u}}_{RBU}} \\ &\qquad + \big[a+b\hat{\mathbf{u}}_{LBD}\cdot\hat{\mathbf{d}}\big]e^{jkr\hat{\mathbf{d}}\cdot\hat{\mathbf{u}}_{LBD}} + \big[a+b\hat{\mathbf{u}}_{RFD}\cdot\hat{\mathbf{d}}\big]e^{jkr\hat{\mathbf{d}}\cdot\hat{\mathbf{u}}_{RFD}}\Big\}\end{aligned} \tag{5.29}$$

We may now apply equation (2.15):

$$\begin{aligned}\tilde{W} = G\frac{1}{a+b}\Bigg\{&\big[a+b\hat{\mathbf{u}}_{LFU}\cdot\hat{\mathbf{d}}\big]\sum_{n=0}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{LFU}\cdot\hat{\mathbf{d}})\,j_n(kr) \\ &+\big[a+b\hat{\mathbf{u}}_{RBU}\cdot\hat{\mathbf{d}}\big]\sum_{n=0}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{RBU}\cdot\hat{\mathbf{d}})\,j_n(kr) \\ &+\big[a+b\hat{\mathbf{u}}_{LBD}\cdot\hat{\mathbf{d}}\big]\sum_{n=0}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{LBD}\cdot\hat{\mathbf{d}})\,j_n(kr) \\ &+\big[a+b\hat{\mathbf{u}}_{RFD}\cdot\hat{\mathbf{d}}\big]\sum_{n=0}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{RFD}\cdot\hat{\mathbf{d}})\,j_n(kr)\Bigg\}\end{aligned} \tag{5.30}$$

Rearranging, we obtain

$$\begin{aligned}\tilde{W} = \frac{Ga}{a+b}\Bigg\{&\sum_{n=0}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{LFU}\cdot\hat{\mathbf{d}})\,j_n(kr) + \sum_{n=0}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{RBU}\cdot\hat{\mathbf{d}})\,j_n(kr) \\ &+\sum_{n=0}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{LBD}\cdot\hat{\mathbf{d}})\,j_n(kr) + \sum_{n=0}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{RFD}\cdot\hat{\mathbf{d}})\,j_n(kr)\Bigg\} \\ +\frac{Gb}{a+b}\Bigg\{&(\hat{\mathbf{u}}_{LFU}\cdot\hat{\mathbf{d}})\sum_{n=0}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{LFU}\cdot\hat{\mathbf{d}})\,j_n(kr) \\ &+(\hat{\mathbf{u}}_{RBU}\cdot\hat{\mathbf{d}})\sum_{n=0}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{RBU}\cdot\hat{\mathbf{d}})\,j_n(kr) \\ &+(\hat{\mathbf{u}}_{LBD}\cdot\hat{\mathbf{d}})\sum_{n=0}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{LBD}\cdot\hat{\mathbf{d}})\,j_n(kr) \\ &+(\hat{\mathbf{u}}_{RFD}\cdot\hat{\mathbf{d}})\sum_{n=0}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{RFD}\cdot\hat{\mathbf{d}})\,j_n(kr)\Bigg\}\end{aligned} \tag{5.31}$$

Now

$$\begin{aligned}\sum_{n=0}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}\cdot\hat{\mathbf{d}})j_n(kr) &= j_0(kr)+j3(\hat{\mathbf{u}}\cdot\hat{\mathbf{d}})j_1(kr) \\ &\quad +\sum_{n=2}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}\cdot\hat{\mathbf{d}})j_n(kr) \\ &= j_0(kr)+j3(\hat{\mathbf{u}}\cdot\hat{\mathbf{d}})j_1(kr)-5P_2(\hat{\mathbf{u}}\cdot\hat{\mathbf{d}})j_2(kr) \\ &\quad +\sum_{n=3}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}\cdot\hat{\mathbf{d}})j_n(kr)\end{aligned} \tag{5.32}$$

where $\hat{\mathbf{u}}$ is any one of $\hat{\mathbf{u}}_{LFU}$, $\hat{\mathbf{u}}_{RBU}$, etc. We may rewrite each of the summations in equation (5.31) in this way; fairly lengthy algebraic and trigonometric manipulation then gives

$$\tilde{W} = \frac{4Ga}{a+b} j_0(kr) + j\frac{4Gb}{a+b} j_1(kr) + F_{\tilde{W}} \tag{5.33}$$

where

$$\begin{aligned}F_{\tilde{W}} = \frac{Ga}{a+b}\Bigg\{&\sum_{n=3}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{LFU}\cdot\hat{\mathbf{d}})j_n(kr)+\sum_{n=3}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{RBU}\cdot\hat{\mathbf{d}})j_n(kr) \\ &+\sum_{n=3}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{LBD}\cdot\hat{\mathbf{d}})j_n(kr)+\sum_{n=3}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{RFD}\cdot\hat{\mathbf{d}})j_n(kr)\Bigg\} \\ +\frac{Gb}{a+b}\Bigg\{&(\hat{\mathbf{u}}_{LFU}\cdot\hat{\mathbf{d}})\sum_{n=2}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{LFU}\cdot\hat{\mathbf{d}})j_n(kr) \\ &+(\hat{\mathbf{u}}_{RBU}\cdot\hat{\mathbf{d}})\sum_{n=2}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{RBU}\cdot\hat{\mathbf{d}})j_n(kr) \\ &+(\hat{\mathbf{u}}_{LBD}\cdot\hat{\mathbf{d}})\sum_{n=2}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{LBD}\cdot\hat{\mathbf{d}})j_n(kr) \\ &+(\hat{\mathbf{u}}_{RFD}\cdot\hat{\mathbf{d}})\sum_{n=2}^{\infty} j^n(2n+1)P_n(\hat{\mathbf{u}}_{RFD}\cdot\hat{\mathbf{d}})j_n(kr)\Bigg\}\end{aligned} \tag{5.34}$$

and contains only spherical harmonic components of order three or higher. These expressions show that

$$A_0\left\{\tilde{W}\right\} \propto aj_0(kr) + jbj_1(kr) \tag{5.35}$$

and that there are no first-order or second-order spherical harmonic components present.

Equation (5.2) may be obtained from equation (5.35) by putting $a = b$.

A similar method may be applied to each of the first-order signals. Taking $\tilde{X}$ as an example, we obtain

$$\tilde{X} = \frac{4G}{\sqrt{3}(a+b)}\left[bj_0(kr) + j3aj_1(kr) - 2bj_2(kr)\right]\cos(q)\cos(f) + F_{\tilde{X}} \qquad (5.36)$$

where $F_{\tilde{X}}$ contains only spherical harmonic components of order two or higher. We therefore have

$$A_{1,1}\{\tilde{X}\} \propto bj_0(kr) + j3aj_1(kr) - 2bj_2(kr) \qquad (5.37)$$

from which equation (5.3) follows by again setting $a = b$.

5.3: Additional B-Format Signal Processing

The first-order soundfield microphone provides facilities to process the derived B-format signals in a number of ways. It is desirable that a second-order soundfield microphone should provide corresponding functionality, so these features are described here.

5.3.1: Rotation & Elevation

The rotation and elevation controls allow the soundfield microphone to be "electronically steered" by modifying the B-format signals in such a way that the new signals correspond to those which would have been obtained had the microphone been differently orientated.

Rotation requires modification only of *X* and *Y*, since *W* and *Z* are independent of azimuth. Let X_1 and Y_1 be the original signals, and X_2 and Y_2 be the modified signals. Then

$$X_2 = \cos(q)X_1 + \sin(q)Y_1 \qquad (5.37a)$$

$$Y_2 = -\sin(q)X_1 + \cos(q)Y_1 \qquad (5.37b)$$

where q is the angle through which the microphone array is effectively rotated. A positive value for q implies an anti-clockwise rotation of the microphone array (which is equivalent to a clockwise rotation of the encoded sound field); hence, setting $q = 90°$ will align the effective centre-front direction with the physical centre-left direction.

Elevation requires only X and Z to be modified:

$$X_2 = \cos(f)X_1 + \sin(f)Z_1 \tag{5.38a}$$

$$Z_2 = -\sin(f)X_1 + \cos(f)Z_1 \tag{5.38b}$$

where ϕ is the required inclination or declination. A positive value for ϕ implies that the effective centre-front direction is elevated above the horizontal plane; therefore, setting $f = 90°$ aligns the effective centre-front direction with the physical centre-up direction.

5.3.2: Side-Fire / End-Fire Switching & Inversion

The soundfield microphone has the common approximate physical form of a cylinder, with the transducer array itself mounted inside one end. It may be operated in either “side-fire” or “end-fire” modes. In side-fire mode, the microphone is positioned vertically with the transducer array uppermost, and the centre-front direction is perpendicular to the axis of the casing. In end-fire mode, the microphone is positioned horizontally and the casing axis is aligned with the centre-front direction.

By default, the first-order soundfield microphone is operated in side-fire mode. When it is instead deployed in end-fire mode, compensatory signal processing is applied so that the output B-format signals are identical to those which would have been obtained had the microphone been operated in side-fire mode. When the microphone is positioned for end-fire operation, it is tipped forward in a 90° rotation about the array y axis; the corrective action required is therefore equivalent to a 90° elevation. Putting $f = 90°$ in equation (5.38) yields

$$X_2 = Z_1 \tag{5.39a}$$

$$Z_2 = -X_1 \tag{5.39b}$$

It is also desirable on occasion to invert the microphone; this is useful, for example, when suspending it from the ceiling of a performance venue, which is a common recording practice [20]. Inversion corresponds to a 180° rotation about the array *x* axis, so that the necessary compensation is a polarity reversal of the *Y* and *Z* signals:

$$Y_2 = -Y_1 \tag{5.40a}$$
$$Z_2 = -Z_1 \tag{5.40b}$$

Although it has been convenient here to describe rotation and elevation before end-fire / side-fire and inversion switching, in practice this switching is applied first; the rotation and elevation controls therefore function identically regardless of which of the three modes the microphone is deployed in.

5.3.3: Dominance

The dominance facility is usually described as a form of "zoom" control; it can be used to modify the encoded sound field in such as a way as to emphasise sources located in a particular direction, and in this sense is somewhat similar to a physical movement of the microphone in that direction [29] [32].

The forward dominance transformation is described by the equations [50] [52]

$$W_2 = \frac{1}{2}\left(l + l^{-1}\right)W_1 + \frac{1}{\sqrt{8}}\left(l - l^{-1}\right)X_1 \tag{5.41a}$$
$$X_2 = \frac{1}{2}\left(l + l^{-1}\right)X_1 + \frac{1}{\sqrt{2}}\left(l - l^{-1}\right)W_1 \tag{5.41b}$$

while *Y* and *Z* remain unchanged. The quantity l is termed the "dominance gain". Let the amplitude, azimuth and elevation of a source in the original encoded sound field be A_1, q_1 and f_1, and let the modified quantities be A_2, q_2 and f_2. It may be shown (see Appendix 3) that the effect of this transformation is to modify the encoded sound field in the following way:

$$A_2 = \frac{1}{2}\left[l\left(1+\cos(q_1)\cos(f_1)\right)+ l^{-1}\left(1-\cos(q_1)\cos(f_1)\right)\right]A_1 \tag{5.42a}$$

$$\cos(q_2)\cos(f_2) = \frac{l^2-1+\left(l^2+1\right)\cos(q_1)\cos(f_1)}{l^2+1+\left(l^2-1\right)\cos(q_1)\cos(f_1)} \tag{5.42b}$$

$$\sin(q_2)\cos(f_2) = \frac{2l\sin(q_1)\cos(f_1)}{l^2+1+\left(l^2-1\right)\cos(q_1)\cos(f_1)} \tag{5.42c}$$

$$\sin(f_2) = \frac{2l\sin(f_1)}{l^2+1+\left(l^2-1\right)\cos(q_1)\cos(f_1)} \tag{5.42d}$$

In the pantophonic case where $f_1 = f_2 = 0$ these reduce to

$$A_2 = \frac{1}{2}\left[l\left(1+\cos(q_1)\right)+ l^{-1}\left(1-\cos(q_1)\right)\right]A_1 \tag{5.43a}$$

$$\cos(q_2) = \frac{l^2-1+\left(l^2+1\right)\cos(q_1)}{l^2+1+\left(l^2-1\right)\cos(q_1)} \tag{5.43b}$$

$$\sin(q_2) = \frac{2l\sin(q_1)}{l^2+1+\left(l^2-1\right)\cos(q_1)} \tag{5.43c}$$

The effect of the forward dominance transformation on a centre-front source is to multiply the amplitude by l, while the angular distance from centre-front for a source anywhere in a small region about the centre-front direction is multiplied by l^{-1}. Note that this means that, while sources near the centre-front direction are emphasised by the increase in amplitude, they are also moved closer to the centre-front axis. In this respect, therefore, the effect of dominance is somewhat different to the effect of physically moving the microphone array; it is nevertheless subjectively acceptable.

Upward dominance can be implemented by replacing X with Z in equation (5.41) and leaving X unchanged. Sideways dominance is similarly possible, but rarely required [32]. As well as being provided as a feature of the soundfield microphone, a dominance control may be included in an ambisonic decoder to allow the listener to achieve this “zoom” effect on playback [32]. Some advanced ambisonic decoders may also make use of a dominance transformation as part of the decoding algorithm [50] [52].

5.4: Synthesis of Stereo Pairs

The soundfield microphone has been described as "... providing output signals equivalent to the outputs which would be obtained from a plurality of coincident microphones" [53]. It has previously been stated that the soundfield microphone may be utilised as a coincident pair for two-channel stereo recording, as well as for B-format recording. This follows from the fact that a linear combination of first-order B-format signals can be found which is equivalent to the output of any desired first-order microphone in any orientation. Clearly, by forming two (or more) different combinations of the B-format signals, the outputs that would have been obtained from two (or more) coincident first-order microphones may be synthesised.

The soundfield microphone offers a number of advantages over other coincident pair microphone arrangements. The much higher frequency to which effective coincidence is maintained is clearly of benefit as far as the quality of the recording is concerned, since phase differences between signals due to microphone spacing are known to degrade both spatial and tonal qualities of recordings [20] [42] [63]. In addition to this, because any possible first-order polar pattern can be obtained from the B-format signals, it follows that a soundfield microphone may act as any possible stereo coincident pair. The two notional microphones may be given any first-order polar pattern, the angle between them may be adjusted, and the pair may be steered and tilted to point in any direction. These adjustments require only a change in the relative contributions of the B-format signals to each synthesised microphone output, and so they may all be performed electronically; that is, remotely and without physically moving the microphone.

It is possible to make a two-channel recording of the outputs from the synthesised coincident pair if that is required. However, if instead the four B-format signals are recorded, then all of the options as described above remain available at the post-session production stage. The choice of stereo pair, with all its aesthetic implications, may then be made at leisure rather than "on the spot" during the perhaps limited time of a recording session or performance. This may be of particular benefit when a live recording is being made away from the studio, since location monitoring facilities are often less than ideal [20] [94].