

**THE UNIVERSITY OF READING**

**On the Theory of the Second-Order Soundfield**  
**Microphone**

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## ABSTRACT

The first-order soundfield microphone currently represents the “state of the art” in coincident microphone array technology for stereo and surround sound recording. The second-order soundfield microphone is the next stage in the development of this technology. This thesis is the first comprehensive presentation of the theory underlying the second-order soundfield microphone, and provides the basic information necessary to facilitate the construction of such a microphone and to characterise its expected performance. As a natural counterpart to this, the theory of first-order pressure gradient microphones is presented in an improved way, and a unified and systematic treatment of second-order pressure gradient microphones, including the proximity effect, is developed. The analysis of ambisonic surround sound reproduction in terms of spherical harmonic matching conditions is extended from the pantophonic to the periphonic case. It is shown that most operations on an encoded soundfield which can readily be performed using first-order ambisonic B-format signals, such as rotation, can be extended to the second-order case; however, the dominance transformation cannot be so extended. Assumptions underlying the definition of the ambisonic B-format signal set in terms of spherical harmonics are examined, and it is argued that this definition may be sub-optimal and result in information loss in some circumstances; an alternative signal set is proposed, which is based directly on partial derivatives of sound pressure, and free from these assumptions and any consequent information loss.

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